Assignment
B.Sc. 4th Semester, 2026
Subject: Mathematics
Paper: Ring Theory & Linear Algebra-I
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1. Let V be the set of sequence $\{a_n\}$ of real numbers. For $\{a_n\}, \{b_n\} \in V$ and any real numbers t, define

$$\{a_n\} + \{b_n\} = \{a_n + b_n\} \ \ and \ \ t\{a_n\} = \{ta_n\}.$$

Prove that, with these operation V is a vector space over $\mathbb{R}$.

2. Let $S = \{(a_1, \ a_2); a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), \ (b_1, b_2) \in S \ and \ c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1 + b_2) = (a_1 + b_1, \qquad a_2 - b_2) \ and \ c(a_1, a_2) = (ca_1, ca_2)$$

Then S is not a vector space with these operations. Justify your answer.

3. Which of the following sets are subspace of $\mathbb{R}^3$ with justification
   (i) $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = 3a_2 \ \ and \ a_3 = -a_2\}$
   (ii) $W_2 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 - 4a_2 - a_3 = 0\}$
4. Determine whether the first polynomial can be expressed as a linear combination of the other two:
   (i) $x^3 - 3x + 5, \ \ x^3 + 2x^2 - x + 1, \ x^3 + 3x^2 - 1$
   (ii) $6x^3 - 3x^2 + x + 2, \ x^3 - x^2 + 2x + 3, \ 2x^3 - 3x + 1$
5. Do the polynomials $x^3 - 2x^2 + 1, \ \ 4x^2 - x + 3 \ \ and \ \ 3x - 2$ generate $P_3(\mathbb{R})$? Justify your answer.
6. Let u, v, w be distinct vectors of a vector space V. Show that if $\{u, v, w\}$ is a basis of V, then $\{u, u + v, u + v + w\}$ is a basis of V.
7. Find bases for the following subspaces of $\mathbb{R}^5(\mathbb{R})$:

$$W_1 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : \ \ a_1 - a_3 - a_4 = 0\}$$

And

$$W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbb{R}^5 : \ \ a_2 = a_3 = a_4 \ \ and \ \ a_1 + a_5 = 0\}$$

What are the dimensions of W_1 and W_2?

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